1

§0 Themes and goals

Themes: 1) Build some theory / practice logical manipulations

Goals: 1) Understand functions,

2) Understand "equivalence"

2

§1 Mathematical relations

Defn: A relation on a set S is a subset $R \subseteq S \times S$.

Notation: $aRb \iff (a,b) \in R$

$\iff$ a is related to b (b by R).

3] e.g. • $S = \{\text{people}\}$

$aRb \iff a$ and b are siblings

• $S = \{\text{animals}\}$

$aRb \iff a$ and b are the same species

• $S = \mathbb{Z}$

$aRb \iff a$ divides b (i.e. $aRb \iff a \mid b$)

4

- $S = \{1, 2, 3\}$

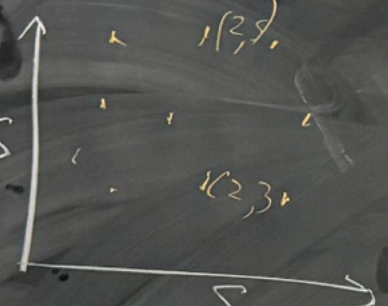
$R = \{(1,1), (2,3)\}$

- $S = \{\text{People}\}$

$a R b \iff$ a and b have the same nationality.

5

Obs: We can picture R:



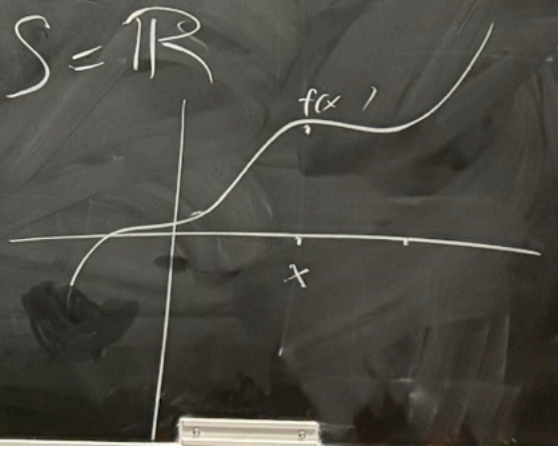
$S \times S$

R

$(1, f(1))$

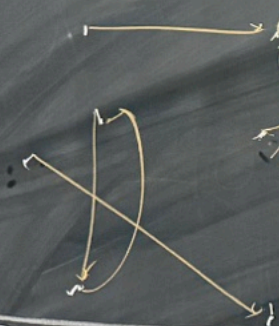
e.g. $f(x)$ $x \in \mathbb{R}$, $S = \mathbb{R}$

$\mathbb{R} \times \mathbb{R} = \{(x, f(x))\}$



6

Obs: We can picture R:



S

$R = \{(a,b) : \text{arrow from } a \text{ to } b\}$

§2 Properties of relations

Defn: A relation R is

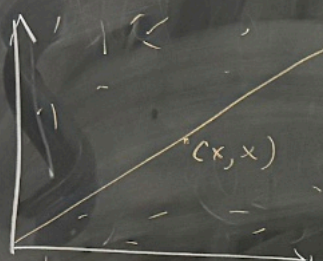
7. • reflexive: if xRx for all $x \in S$

• symmetric: $aRb \Longrightarrow bRa$

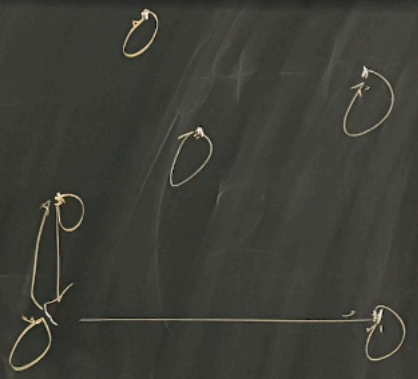
• transitive: $aRb, bRc \Longrightarrow aRc$

8.

Reflexive:



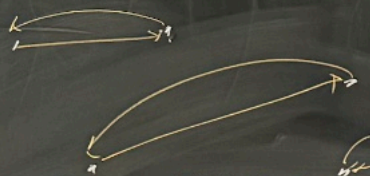
All loops at a single dot exist



Symmetric:

Whole diagonal contained in picture

Symmetric flipping over digonal



If an arrow exists, then the opposite orientation arrow exists

Not as clear

head of the arrow meets tail of other

If 2 sides of a triangle exist then the full triangle exists

Q What are examples of R satisfying 2 of 3?

A

✓R	xR	y
✓S	if $\lvert x-y\rvert \leq 1$	
xT	$x, y \in \mathbb{R}$	

xR	Not possible
✓S	$x \sim y \overset{\text{Symm}}{\Rightarrow} y \sim x$
✓T	Trans $\Rightarrow x \sim x$

✓R a|a

xS a|b $\not\Rightarrow$ b|a 2|6, 6|2

✓T a|b, b|c $\Rightarrow$ a|c